WARM UP

a) Calculate the average rate of change for the function $f(x) = 2x^3 - 3x^2$ **on the interval** x **= -1 to** x **= 3.**

b) Calculate the slope of the tangent line for the same function where $x = 2$ **.**

$$AROC = \frac{f(x_1) - f(x_2)}{x_1 - x_2}$$

$$\frac{f(-1) - f(3)}{-1 - 3}$$

$$\frac{-5 - 27}{-4}$$

$$\frac{-32}{-4}$$

$$= 8$$

$$f(-1) = 2(-1)^3 - 3(-1)^2$$
$$= -5$$

$$f(3) = 2(3)^3 - 3(3)^2$$
$$= 27$$

$$\frac{f(1.99) - f(2)}{1.99 - 2}$$

$$\frac{3.880898 - 4}{-0.01}$$

$$\frac{-0.119106}{-0.01}$$

$$11.9$$

$f(1.99)$

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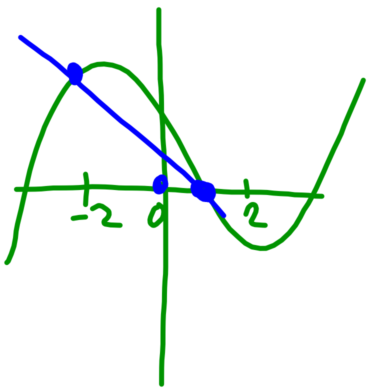
Calculus 120
Unit 1: Rate of Change and Derivatives

February 4, 2019: Day #4

1. Quick quiz tomorrow - AROC and IROC

2. Assignment Due today



find AROC from [-2,0]

Jan 9-1:43 PM

Curriculum Outcomes

C1. Explore the concepts of average and instantaneous rate of change.

Limits of Rates of Change

http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_avg_ROC.html

Remember, instantaneous rates of change are slopes of tangent lines at particular points.

Essentially, the method we used over the last couple of days is a limiting process. Our interval gets smaller and smaller. Our "x_1" value approaches the "x_2" value (x_2 being the point at which we want to know IROC). Now that we know how to calculate limits, we can calculate exact values of instantaneous rates of change by finding the limit of our slope.

Feb 2-6:30 PM

Instantaneous rates of change (slopes of tangent lines) can be found by using the following formula...

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

OR

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

"a" is the x value at which we want to calculate the IROC.

Ex: Find the instantaneous rate of change of $y = 2x^2 + 4x - 1$ when x = 2. "a"

$$\lim_{x \to 2} \frac{f(x) - f(2)}{x-2}$$

$f(2) = 2(2)^2 + 4(2) - 1$
$= 8 + 8 - 1$
$= 15$

$$\lim_{x \to 2} \frac{2x^2 + 4x - 1 - 15}{x-2}$$

$$\lim_{x \to 2} \frac{2x^2 + 4x - 16}{x-2}$$

$$\lim_{x \to 2} \frac{2(x^2 + 2x - 8)}{x-2}$$

$$\lim_{x \to 2} \frac{2(x+4)(x-2)}{x-2}$$

IROC $= 2(2+4)$
$= 2(6)$
$= 12$

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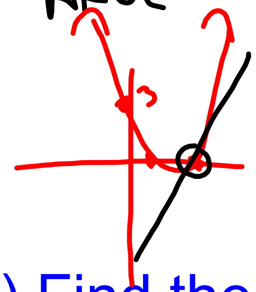
Ex: a) Find the average rate of change of $f(t) = t^2 - 4t + 3$ from $t = 3$ to $t = 5$

$AROC = \frac{f(3) - f(5)}{3 - 5}$

$f(3) = 3^2 - 4(3) + 3$
$= 0$
$(3, 0)$



b) Find the instantaneous rate of change at $t = 3$

$\lim_{t \to 3} \frac{f(t) - f(3)}{t - 3}$

$\lim_{t \to 3} \frac{t^2 - 4t + 3 - 0}{t - 3}$

$\to \lim_{t \to 3} \frac{(t-3)(t-1)}{t-3}$

$IROC = 3 - 1$
$= 2$

Slope of tangent line!!

c) What is the equation of the tangent line at $t = 3$?

$m = 2$ Point $(3, 0)$

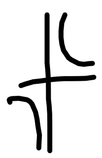
$y = mx + b$

$0 = 2(3) + b$
$0 = 6 + b$
$-6 = b$

$y = 2x - 6$

Feb 2-6:35 PM

Determine the equation of the tangent line to the hyperbola $y = \frac{1}{x}$ at the point (-2, -1/2).



Feb 2-6:36 PM

A spherical balloon is being inflated. Find the rate of change of the volume with respect to radius when the radius is 10 cm. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$**.**

$$\lim_{r\to 10} \frac{f(r) - f(10)}{r-10}$$

$V = \frac{4}{3}(\pi)10^3$

$$\lim_{r\to 10} \frac{\frac{4}{3}\pi r^3 - \frac{4}{3}\pi(10)^3}{r-10}$$

$$\lim_{r\to 10} \frac{\frac{4}{3}\pi(r^3 - 10^3)}{r-10}$$

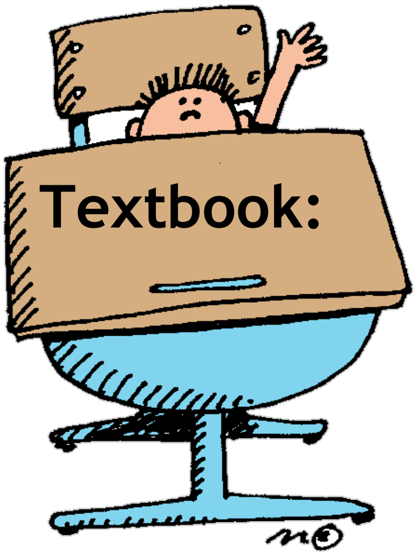
$$\lim_{r\to 10} \frac{\frac{4}{3}\pi (r-10)(r^2 + 10r + 100)}{r-10}$$

$$= \frac{4}{3}\pi(10^2 + 10(10) + 100)$$

$$= \frac{4}{3}\pi(300)$$

$$= \frac{1200\pi}{3}$$

$$= 400\pi \text{ cm}^3/\text{cm}$$



Practice

Page 92-93 #1a, 5a, 9, 11
23, 27, 29, 31

Feb 7-9:29 AM

Attachments

- 2.1_74_AP.html
- 2.1_74_AP.swf
- 2.1_74_AP.html